

General Certificate of Education
June 2008
Advanced Level Examination



MATHEMATICS
Unit Further Pure 3

MFP3

Monday 16 June 2008 1.30 pm to 3.00 pm

For this paper you must have:

- a 12-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP3.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

- 1 The function $y(x)$ satisfies the differential equation

$$\frac{dy}{dx} = f(x, y)$$

where

$$f(x, y) = \ln(x + y)$$

and

$$y(2) = 3$$

Use the improved Euler formula

$$y_{r+1} = y_r + \frac{1}{2}(k_1 + k_2)$$

where $k_1 = hf(x_r, y_r)$ and $k_2 = hf(x_r + h, y_r + k_1)$ and $h = 0.1$, to obtain an approximation to $y(2.1)$, giving your answer to four decimal places. (6 marks)

- 2 (a) Find the values of the constants a , b , c and d for which $a + bx + c \sin x + d \cos x$ is a particular integral of the differential equation

$$\frac{dy}{dx} - 3y = 10 \sin x - 3x \quad (4 \text{ marks})$$

- (b) Hence find the general solution of this differential equation. (3 marks)

- 3 (a) Show that $x^2 = 1 - 2y$ can be written in the form $x^2 + y^2 = (1 - y)^2$. (1 mark)

- (b) A curve has cartesian equation $x^2 = 1 - 2y$.

Find its polar equation in the form $r = f(\theta)$, given that $r > 0$. (5 marks)

- 4 (a) A differential equation is given by

$$x \frac{d^2y}{dx^2} - \frac{dy}{dx} = 3x^2$$

Show that the substitution

$$u = \frac{dy}{dx}$$

transforms this differential equation into

$$\frac{du}{dx} - \frac{1}{x}u = 3x \quad (2 \text{ marks})$$

- (b) By using an integrating factor, find the general solution of

$$\frac{du}{dx} - \frac{1}{x}u = 3x$$

giving your answer in the form $u = f(x)$. (6 marks)

- (c) Hence find the general solution of the differential equation

$$x \frac{d^2y}{dx^2} - \frac{dy}{dx} = 3x^2$$

giving your answer in the form $y = g(x)$. (2 marks)

- 5 (a) Find $\int x^3 \ln x \, dx$. (3 marks)

- (b) Explain why $\int_0^e x^3 \ln x \, dx$ is an improper integral. (1 mark)

- (c) Evaluate $\int_0^e x^3 \ln x \, dx$, showing the limiting process used. (3 marks)

- 6 (a) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 2 \frac{dy}{dx} - 3y = 10e^{-2x} - 9 \quad (10 \text{ marks})$$

- (b) Hence express y in terms of x , given that $y = 7$ when $x = 0$ and that $\frac{dy}{dx} \rightarrow 0$ as $x \rightarrow \infty$. (4 marks)

Turn over for the next question

Turn over ►

7 (a) Write down the expansion of $\sin 2x$ in ascending powers of x up to and including the term in x^3 . (1 mark)

(b) (i) Given that $y = \sqrt{3 + e^x}$, find the values of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ when $x = 0$. (5 marks)

(ii) Using Maclaurin's theorem, show that, for small values of x ,

$$\sqrt{3 + e^x} \approx 2 + \frac{1}{4}x + \frac{7}{64}x^2 \quad (2 \text{ marks})$$

(c) Find

$$\lim_{x \rightarrow 0} \left[\frac{\sqrt{3 + e^x} - 2}{\sin 2x} \right] \quad (3 \text{ marks})$$

8 The polar equation of a curve C is

$$r = 5 + 2 \cos \theta, \quad -\pi \leq \theta \leq \pi$$

(a) Verify that the points A and B , with **polar coordinates** $(7, 0)$ and $(3, \pi)$ respectively, lie on the curve C . (2 marks)

(b) Sketch the curve C . (2 marks)

(c) Find the area of the region bounded by the curve C . (6 marks)

(d) The point P is the point on the curve C for which $\theta = \alpha$, where $0 < \alpha \leq \frac{\pi}{2}$. The point Q lies on the curve such that POQ is a straight line, where the point O is the pole. Find, in terms of α , the area of triangle OQB . (4 marks)

END OF QUESTIONS